UNIVERSITY OF NOTRE DAME DU LAC

NOTRE DAME, INDIANA

The Notre Dame Mathematics Teacher Training Program

Summer, 1959

Duration: June 19 - August 4, 1959

The Notre Dame Mathematics Teacher

Training Program

1959

I The Student Body

It has been generally recognized for a long time that there exists a pressing need for a happy medium between two extremes in the training of mathematics teachers - the one extreme emphasizing the methods of teaching to the exclusion of the consideration of the mathematical content, and the other extreme allowing an early preoccupation with highly technical detail of some specialized mathematical subject with the consequent neglect of the broader mathematical culture so essential for the effective teaching of mathematics.

The present summer teacher training program at Notre Dame is an attempt to provide the needed happy medium. It began in 1947 with a small group of high-school and junior-college teachers. The student body has grown steadily since then. In the summer of 1956, there were 84 graduate students in the mathematics department (all of whom were practicing high-school or college teachers) working toward an M.S. degree in mathematics and 22 students in the related science programs who took some of the courses in the mathematics teacher training program.

A new impetus to the development of our program has been provided by the National Science Foundation Grant for High-School Teachers for the summers of 1957 and 1958. In the summer of 1957, there were 164 graduate students in mathematics (most of whom were practicing high-school teachers) - many of these working toward an M.S. degree in mathematics. Eighty-four (84) of these graduate students were high-school teachers supported wholly or in part by the National Science Foundation. In the summer of 1958, the number of students participating in the Teacher Training Program totalled 229 (almost all of these again were practicing

high-school teachers). Of these, one hundred and fifty one (151) were high-school teachers supported wholly or in part by the National Science Foundation.

With the help of the National Science Foundation, we once again hope to be able to make it possible for a significant number of high-school teachers to participate in our program. We expect the enrollment in the summer of 1959 to be about 250.

Many of the teacher-students in our program come in order to pursue systematically for five summers either the program outlined in detail in the section on curriculum or a combination of this program with similar programs of study leading to an M. S. in Mathematics with a possible minor in some related field. Seven teachers completed their work for an M. S. in 1957 and 16 teachers completed their M. S. work in 1958.

We do have a number of students who come only for a summer or two to listen to lectures in some topic or topics of immediate importance to them in their teaching.

Now and then in our summer teaching we come across students of exceptional mathematical ability who would not have come into contact with advanced mathematical ideas outside of a summer program such as ours. We encourage such students to come during the regular academic year and to pursue a much more intensive course of studies leading to a higher degree - often a Ph. D. degree.

At times visitors with more advanced training come to observe and to comment upon our work in teacher training. We welcome such visitors and hope that their number will increase in the future. We have been much encouraged by the sympathetic interest in our work shown by such visitors in the past.

II Duration

Our Mathematics Teacher Training Program takes place during the

seven-week summer-school session at Notre Dame. In 1959, our summer session will start on June 19, and will end on August 4. Registration will take place on June 19.

III Objectives

It is the aim of the Mathematics Teacher Training Program to bring the high school and college teachers into close contact with the best traditions of mathematics and of the teaching of mathematics. We believe that if teachers succeed in assimilating such traditions through an effective program of thoughtful mathematical instruction, then they will perpetuate these same traditions in their own teaching.

Our mathematics courses in the Teacher Training Program are not courses in the methods of teaching mathematics. Our courses contain a carefully prepared introduction to those ideas of higher mathematics which are of particular value to the teacher of mathematics.

We feel that our Teacher Training Program must fulfill two conditions: First, through an enlightened realistic plan of training it should develop in our students greater competence in their chosen field and through that, greater confidence in themselves and greater interest in their work. Second, in order to be truly successful, our curriculum should represent the best mathematical thought of our time. Toward this end, we are making a determined effort to interest outstanding mathematicians in our work and in coming here during the summer to teach what they would consider a good teachers' course in their chosen fields.

IV (1) The Curriculum

Students without advanced mathematical training are advised to follow closely the plan outlined below. For students with some experience in mathematics a suitable modification is worked out upon consultation.

First Summer

Mathematics 123s - Elementary Number Theory.

The algebra of classes. Basic properties of the ring of integers, divisibility, Euclid's Algorithm, prime numbers, factorization into primes, congruences. Diophantine equations. Congruences with one unknown.

In addition to the usual basic concepts of set theory the content is approximately that of Chapters I-VII of Uspensky and Heaslet's "Elementary Number Theory".

Mathematics 228s - Higher Algebra I.

Basic mathematical concepts (mappings, relations, etc.). Groups, Rings, Integral Domains, Fields. Polynomial rings and their fundamental properties.

The level of precision is approximately that of Marie Weiss's "Higher Algebra for the Undergraduate".

In teaching our "First-Summer" students, we set as our principal objectives the development of the precise use of language, the correct use of the processes of mathematical reasoning, and, last but not least, some appreciation of the art of mathematical discovery and of problem solving. A strong effort is made through the use of numerous examples and exercises to give the student an adequate background for the understanding of abstract mathematical concepts and the student is encouraged to participate in the problem seminars related to the courses.

Second Summer

Mathematics 229s - Higher Algebra II.

Vector spaces. Matrices and linear transformations. Determinants, volumes, and systems of linear equations.

The level of precision is approximately that of Schreier and Sperner's "Einfurung in die Analytische Geometrie und Algebra".

Mathematics 281s - Geometry I.

A critical re-examination of the fundamental concepts of geometry.

Professor G. Y. Rainich, Summer, 1957.

New material in vector algebra, projective and non-Euclidean geometry is introduced on the basis of and in close contact with elementary geometry which is reviewed and refined in the process. The axiomatic point of view is mentioned, but not insisted upon and the theoretical work is accompanied by much work involving geometrical constructions and vector computations.

Although in the case of the "Second-Summer" student it is possible to teach more difficult mathematical ideas by building upon the foundations laid during the student's first summer at Notre Dame, still much more emphasis must be placed upon the consolidation of the gains in the mastery of the fundamentals achieved during the first summer.

The geometry course offers a particularly fine opportunity for the refinement of the student's mathematical perception.

Third Summer.

Mathematics 282s - Geometry II.

Continuation of Geometry I. The level of treatment approximates that of Veblen and Young, Vol. I.

Mathematics 140s - Analysis I.

Real Number System. Functions. Sequences. Limits. Continuity.

Fourth Summer

Mathematics 250 - Analysis II.

Fundamental Concepts of Calculus.

Level of precision of Analysis I and II approximately that of Landau "Differential and Integral Calculus".

Mathematics 253 - Irrational Numbers.

Professor H. D. Kloosterman, Summer, 1957

It is the objective of this course to make the student feel at home with the basic ideas and methods of Analysis by bringing him into contact with individual (and famous) examples of irrational numbers and by giving him an opportunity of working with a variety of special methods of approximation of irrationals by rational numbers (such as g-adic expansions, Cantor expansions, continued

fraction expansions, and infinite products). As a by-product the student comes into contact with a rich sample of good mathematical ideas in Number Theory and algebra as well as Analysis.

Fifth Summer

Mathematics 234 - Combinatorial Problems.

Professor Thoralf Skolem, Summer, 1958

Starting from the combinatorial notion of operation tables, in particular Latin squares, the properties of binary operations of finite systems were studied. The structure of finite groups, rings and fields was discussed, and properties of these structures not commonly discussed in the basic modern algebra courses were considered. Problem sets developed a very wide variety of isomorphic representations of finite structures.

The above discussion was followed by a brief sketch of algebraic number fields, and concepts of integers, primes, units, norms in algebraic number fields and in the algebra of quaternions.

Durichlet's principle (Schubfachschluss) was introduced and a series of applications made of it to obtain upper bounds for least non-trivial solution in integers of systems of linear equations or congruences. This was again used to express positive integers as sums of squares and to solve in integers the homogeneous ternary quadratic equation. Further, Durichlet's principle was used in the study of Pell's equation, in particular the proof of a theorem which makes the solution

of some exponential equations possible.
At last the triple systems of Steiner and similar conbinatorial subjects were treated.

Professor S. Chowla,
Summer, 1958.

Mathematics 335 - Elementary Analytic Number Theory

IV (2) Lecture Series.

Lecture Series (a). Curriculum.

In the summer of 1959, we plan to continue the series of lectures and seminars devoted to the discussion of experimental high-school curricula and experimental undergraduate college curricula.

We have invited people deeply concerned with the problems of revision of high-school curricula and with the related problem of Teacher Training, to visit us and to explain their ideas. In 1957, we were very fortunate to have with us Dean Alfred E. Meder, Jr., Executive Director, Commission on Mathematics, College Entrance Examination Board, and three members of the "Illinois Group", Professors Max Beberman, Director, University of Illinois Commission on School Mathematics, Norman T. Hamilton and Joseph Landin. In 1958, Professor Max Beberman gave a series of very interesting lecture-demonstrations under the title "This-is-how-I-do-it". Professor Herbert Vaughan, in addition to a series of ten lectures on logic, discussed the Illinois Project of high-school textbook writing and reported on the work of the Yale textbook writing group. Professor W. Warwick Sawyer discussed the importance of a creative approach to teaching and the possible role his "Mathematics Student Journal" may play as a useful teacher's aid and as a tool for the discovery and encouragement of gifted high-school students. Professors Joseph Landin and Norman Hamilton have returned to discuss the problems and the aims of the Academic Year Institutes and in particular to report on their work with such an Institute at Illinois.

Lecture Series (b). Teaching aids

We were able to begin our Lecture Series (b) in the Summer of 1958. Our experiment consisted of a series of nine (9) lectures over closed circuit TV to the group of over 250 participating mathematics teachers and other interested students and members of the University Staff. The program was viewed over nine (9) television receivers. Members of the TV audience came to the studio on a rotation plan so that almost everyone was able to get a close look at the mechanics of the broadcasts.

There was a round table discussion on the lessons of this experiment and an evaluation of the written comments is planned for the near future.

The planning of our TV experiment began in 1958 when Reverend Edmund Joyce, our Executive Vice-President asked us to experiment with closed circuit television and to study its educational possibilities in connection with the teaching of large groups of freshmen and sophomores. At the time we felt very unhappy about the prospect of trying out a new medium of communication on a large group of young people as unformed and as inarticulate as college freshmen are by and large.

After some soul searching, we came to the conclusion that the participants of our Teacher Training Program would be a more appropriate audience for a TV experiment. It seemed that they would be aware of the problems involved in changing the student-teacher relationship through the use of the new medium and that even if the experiment would be a failure as a teaching venture, it would still provide significant experience for the partcipating teachers. It was our good fortune that the ideas in the lectures did come across to the student even though we did not arrange for a closer contact with the audience to compensate for the more remote position of the TV teacher. One should note however that for the 120 beginners in the Mathematics Teacher Training Program some opportunity for discussion in smaller groups was provided by the seminars in the related number theory course. The interest in the whole experiment was exceedingly keen, for, as it appeared later, some of the teachers expected

to be involved in teaching over closed circuit TV and most of them read about experiments involving such teaching on the high-school level and expected to be involved in such teaching sooner or later.

As the subject matter we chose the algebra of classes which we have used for some time as introductory material for our summer group not only because of its fundamental importance in mathematics but also because, it seems to furnish a natural environment for the discussion of the problem of the communication of ideas, of the role of (technical) language, of the position of a deductive abstract theory versus its concrete realizations, and last (but not least) of the importance of the heuristic considerations, of observation and experiment both in the formation of an abstract theory as well as in the exploration within the confines of such an abstract theory.

Although one tried to foresee the difficulties which the new medium would present to the TV teacher, one was completely unprepared for the full impact of the new impressions. Of these I would single out the vivid realization of how dependent the teacher becomes upon the continual two-way flow of information in the classroom and the critical dependence of the lecturer upon the TV director (in my case the very able and very patient Bill Mackenzie) and his crew.

Our experiment seems to indicate that TV can become one of the important tools of education and that there is much one must learn through intelligent experimentation and a thoughtful study of the basic problems posed by the new medium. Even our limited experience seems to indicate that it is not wise to transmit the average classroom routine unchanged on to a TV screen. It is a folly to assume that what is unimaginative and dull in the classroom, becomes brilliant and inspiring when transferred to the TV screen. If anything, the new medium is more sensitive than the old one to transgressions against good taste and intellectual integrity.

One of the real dangers of TV is revealed by its very designation as the new mass medium of communication. Among the most important concerns of education is the discovery and nurture of talent and the provision of a favorable climate for creative work. One must guard against the danger that easy access to the many may encourage the neglect of the gifted few.

A potentially important use of TV outside of the confines of the university campus is suggested by our experience in working with the high-school teachers in the environs of Notre Dame and with their gifted pupils. I have in mind a program which would involve two one hour broadcasts a week and in which we would cooperate with the teachers working within the TV radius of Notre Dame in providing courses for gifted high-school students.

We hope to be able to continue and to expand our TV experiment in the summer of 1959.

Lecture Series (c). Creative Mathematics

We plan to continue our program of lectures and problem seminars under the title "Creative Mathematics". Usually, each series of five to ten lectures is to be self-contained and is to discuss some research problem accessible by elementary methods. The seminar will be devoted to the discussion of problems on the intermediate and advanced levels. The aim of this program is to emphasize the living - the dynamic nature of mathematics. Visiting lecturers for 1957 in this program were Professor E. J. McShane, University of Virginia and Professor Alfred Brauer, University of North Carolina. Visiting lecturers for 1958 were Professor E. J. McShane (June 25, 26), Professor A. A. Albert (July 7 - July 11), and Professor W. Warwick Sawyer (July 19, 20). Professor Sawyer discussed ways in which the viewpoint of discovery may be brought down directly into high-school teaching.

IV (3) Our Visitors

It is clear that the first two summers are the most critical. During this period the students are taught by members of our own staff or by visitors especially interested in the beginner's problems.

By his fourth summer, each student will have had one summer of number theory, two summers of higher algebra, two summers of geometry, and two summers of mathematical analysis. By this time, an interested student is ready to delve more deeply into a subject of his choice and to derive benefit from more sophisticated mathematical courses. In particular, by then the student is ready to benefit by the stimulating contact with the teaching of distinguished mathematical visitors.

In the summer of 1957, we were again very fortunate to have with us Professor H. D. Kloosterman of the University of Leiden whose interesting course entitled "Irrational numbers" was highly successful and produced a deep impression on the students. Professor G. Y. Rainich of the University of Michigan was the second distinguished visitor-member of our staff. His course in geometry and the related seminars provided an unusual and thought provoking introduction to axiomatic geometry and evoked a very warm response from our student-teachers.

In 1958, our visiting staff consisted of Professor David Lipsich of the University of Cincinnati, Professor John Riner of St. Louis University, Professor R. P. Bambah of Panjab University, Professor S. Chowla of the University of Colorado and the Institute for Advanced Study, Professor Thoralf Skolem of the University of Oslo, Norway, and Professor Herbert Vaughan of the University of Illinois. Professor Riner participated in our basic modern algebra program, Professors Bambah and Lipsich assumed the responsibility for the basic analysis program and Professors Chowla and Skolem taught topics courses which provided intellectual challenge for the more experienced participants. Professor Vaughan gave a series of 10

lectures serving as an introduction to a formal study of mathematical logic.

We were fortunate in the past in having a number of very accomplished mathematicians come and give one and sometimes several lectures to our students in the Teacher Training Program. Of these one may mention Professor Richard Brauer, Professor Paul Erdos, Professor Solomon Lefschetz and Professor Harry Vandiver.

Our visitors in 1957 were Professor E. J. McShane, University of Virginia, and Professor Alfred Brauer of the University of North Carolina. In 1958, our visitors included Professor A. A. Albert of the University of Chicago, Professor Max Beberman of the University of Illinois, Professor Ram Behari of the University of Delhi, Professor Joseph Landin of the University of Illinois, Professor E. J. McShane of the University of Virginia, Professor Hans Rohrbach of the University of Mainz, Professor W. W. Sawyer of Wesleyan University, Professor Sigmund Selberg of Norwegische Technische Hochschule, Trondheim, Norway and Professor Hans Zassenhaus of McGill University.

With particular reverence and appreciation one recalls the summer term when the late Professor Max Dehn taught a course in Projective Geometry in our program.

IV(4) The Staff

Our 1958 staff consisted of Reverend Ferdinand Brown, C.S.C. (Associate Director), Sister Barbara Ann Foos, Brother James Gray, Professors Haaser, Jeglic, Lewis, Nastucoff, Otter, Ross (Director of the Program), Sullivan, Taliaferro and Weinstock, and Visiting Professors Bambah, Chowla, Skolem, Lipsich and Riner.

We expect that most permanent members of the Notre Dame Staff who taught in the summer of 1958 will again teach in the summer of 1959, and we look forward to the return of some of our visitors.

V Academic Credit

Each course meets five times a week, Monday through Friday for a full clock hour and carries the credit of three semester hours.

Stipend holders in the National Science Foundation Summer Institute do not pay tuition and are entitled to credit for each successfully completed course, provided that their interest in such credit is indicated at registration time.

For students with appropriate qualifications not holding a National Science Foundation stipend academic credit is available up to 6 semester credit hours at the cost of $ 130 in tuition, and up to $20 in other fees and costs (Lab fees, textbooks). The degree of an M. S. in mathematics can be earned upon the successful completion of ten courses (30 semester hours of credit).

VI Counseling

In 1957, as an extension of our counseling program we were able to arrange fourteen (14) problem seminars for students in our basic courses such as Number Theory, Higher Algebra I and Geometry I. We increased the number of problem seminars in the summer of 1958 to nineteen (19).

A very great emphasis is laid upon the counseling of students.

VII Facilities

A. A first rate mathematics and science library. The Mathematics Library is housed in the Nieuwland Science Building in a well-equipped reading room with a professionally trained library staff for assistance to students and faculty in library reference work. The Mathematics Library has a good working collection both for undergraduate instruction and for research.

   Special attention has been given to the collection of mathematical research journals from many countries and in many languages. Not only are most of the currently published journals received but back files of all the important journals are maintained.

B. A modern bookstore which sponsors large textbook exhibits.

C. Modern classrooms.

D. The campus itself is practically a park with two lakes, shaded areas, walks, etc. Recreation is available for men on the golf course, swimming in the lake, tennis courts, etc. Free concerts, etc.

E. Housing is ample on campus for both men and women students.

F. Housing for married couples can be arranged in South Bend.

G. The University cafeteria is open to the public from 7:30 a.m. to 7:00 p.m. each day.

H. Facilities for informal social gatherings such as faculty-student teas.

The foregoing is a bare outline of the activities of our Teacher Training Program. It is difficult to describe in words the spirit which is created through the working together of an eager, interested, and devoted student body and a dedicated faculty.

Arnold E. Ross

Arnold E. Ross, Head
Department of Mathematics and
Director of the Program