Ross Rectangle Problem

Given a line segment from the origin O to a point P, a geometer constructs a rectangle of area 1 with the segment OP as base, and oriented counter-clockwise from the segment.

Starting from $P_0 = (1, 0)$ on the x-axis, she draws a square with base OP_0. (That square is red in the picture.) With "diagonal point" $P_1 = (1, 1)$, she draws an area 1 rectangle on base OP_1 (blue in the picture). With diagonal point $P_2 = (\frac{1}{2}, \frac{3}{2})$, she draws the next rectangle with base OP_2 (green).

Continue the process: Using diagonal point P_n of the n^{th} rectangle, construct the next rectangle with base OP_n, area 1, and counterclockwise from OP_n.

(a) Let Q_n be the fourth corner of the n^{th} rectangle. The picture indicates that Q_0 lies on segment Q_1P_2, and Q_1 lies on segment Q_2P_3.

Does Q_n lie on the the segment $Q_{n+1}P_{n+2}$, for every $n = 0, 1, \ldots$?

If so, provide an explanation (proof) of why that happens.

(b) Let $b_n = |OP_n|$ be the base length of the n^{th} rectangle. Then the adjacent side has length $|OQ_n| = 1/b_n$, because the area is 1. For instance, $b_0 = 1$, $b_1 = \sqrt{2}$, ; and $b_3 = \sqrt{5/2}$.

- Show that the lengths b_n grow without bound as n increases. How fast does the sequence (b_n) grow? As $n \to \infty$, is b_n growing linearly in n? Or is it more like n^r for a constant r in $(0, 1)$? Or perhaps like $\log n$?
- Do the points P_n spiral repeatedly around O as n increases?