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## Problem (from the 2014 Ross Program Application)

My calculator has the usual keys for arithmetic and for “memory.” After I dropped it in the bathtub, several keys (like  $\boxed{\times}$  and  $\boxed{\div}$ ) don’t work at all. The only keys that work are the number keys, parentheses keys, store and recall memory keys, and the following three operation keys:

$$\begin{array}{ccc} \boxed{+}, & \boxed{-}, & \text{and} & \boxed{x^{-1}}. \\ \text{(plus key)} & \text{(minus key)} & & \text{(reciprocal key)} \end{array}$$

Can you use that broken calculator to compute products?

That is, if  $A$  and  $B$  are stored in calculator memories, can the product  $AB$  be computed using only those few working keys? Stated in another way:

**Question.** Is there a formula for  $AB$  in terms of  $A$ ,  $B$ , parentheses, and those three operations? Justify your answer.

Note: a formula is required, so “repeated addition” is not a valid way to calculate  $AB$ .